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MODAL RESPONSE SUMMATION FOR SEISMIC QUALIFICATION[a]

By David W. Lindley[1], and Dr. J. Roland Yow[2]

INTRODUCTION

Nuclear power plant equipment is usually classified in the initial phase of seismic analysis as either rigid or flexible. Equipment is considered flexible if the magnitude of the foundation motion can be magnified by resonant modal response of the equipment. It is considered rigid if all the natural frequencies of vibration are greater than the resonant range of the foundation motion.

Rigid equipment can be analyzed using the peak acceleration of the foundation motion in a static analysis. However, flexible equipment requires a dynamic analysis to determine the magnitude and interaction of the various modal responses. The detailed computation required for a dynamic analysis can be quite costly. Often, an approximate method, Response Spectra Modal Analysis, is substituted.

This paper examines the use of Response Spectra Modal Analysis and presents the analytical basis for a modification intended to reduce the computing costs and increase the validity of the results.

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In a Response Spectra Modal Analysis, phasing relationships among the various natural modes of vibration are not determined. Only the peak response of each mode is determined. Since all modes may not peak at the same time, the maximum response of the structure must be approximated by a combination of the peak modal responses.

The most common method used to combine the peak modal responses is by square root of the sum of the squares, SRSS. Closely spaced modes, (modes with frequencies within ten percent of each other), are combined absolutely and included as a single term in the SRSS summation. This method is statistically valid if each of the responses peaks randomly in time. The responses of rigid modes violate this criterion because they all tend to peak at the time that the base excitation peaks.

Investigations by Gwinn and Waal (3) show that SRSS responses of nearly rigid structures are not conservative compared to time history responses. Biswas and Duff (2) further demonstrate that SRSS responses close to support points are lower than time history responses. The modified response summation, MRS, presented in this paper improves the SRSS approximation by allowing for simultaneous rigid response.

A second approximation is introduced by Response Spectra Modal Analysis if all natural modes of vibration are not included in the summation. The costs of processing mode shapes can be reduced if only "significant" modes are included in the summation. However, no widely accepted method for determining the significance of a mode has been established. The choice of modes is left to the engineer's discretion. In the modified response summation, only flexible modes need be included in the summation. The SRSS approximation is improved by reducing the possibility of overlooking significant modes.

A typical earthquake acceleration response spectrum is shown in Figure 1. The spectrum shows that for high frequencies, the acceleration response becomes constant. This constant is known as the zero period acceleration, ZPA, and is equal to the peak base acceleration. The spectrum frequency where the response is first considered constant is known as the cut-off frequency. Modes with frequencies above the cut-off frequency are considered rigid. Modes below the cut-off frequency are flexible.

The modified response summation being proposed by this paper utilizes a static analysis based on zero period acceleration. The structural response represented by the static analysis is indicated by the dashed area in Figure 1. The static analysis, in effect, applies the peak base acceleration to every mode of the structure. The remaining structural response results from the unused part of the flexible responses.

The MRS combination rule is given by

$$R = [R_s^2 + (\sum_1^{\ell} R_d^2)]^{1/2} \tag{1}$$

where:

R = the total response of the structure,

R_s = the response of the structure to a static analysis based on ZPA acceleration,

R_d = the dynamic response of a flexible mode based on the square root of the difference between the response spectra acceleration, squared, and the ZPA acceleration, squared;

ℓ = the number of flexible modes.

The first term of equation (1) represents the peak response due to absolute acceleration of the base. The second term approximates peak responses of

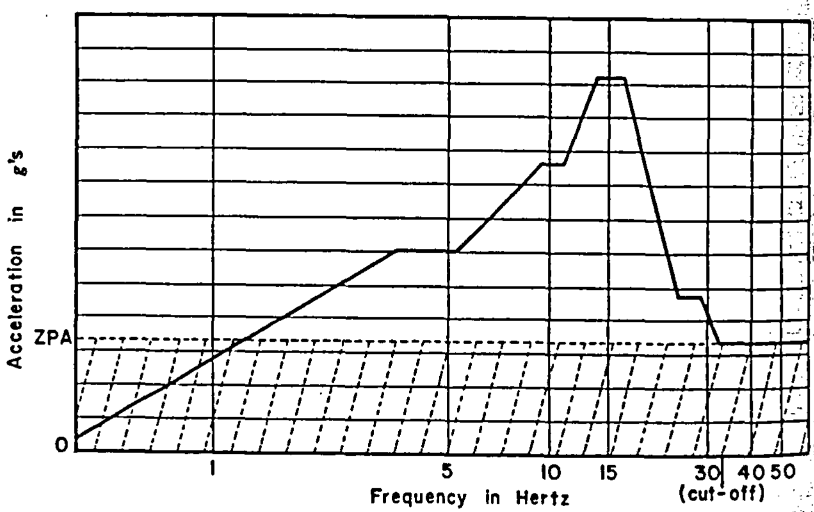

FIG. I.— Typical Broadened Acceleration Response Spectra.

flexible modes due to accelerations relative to the base. Since the absolute and relative accelerations need not peak simultaneously, an SRSS combination is used in equation (1).

In the limiting case of no flexible modes, equation (1) is reduced to a simple static analysis. This complies with the normal method used for analyzing rigid equipment. In the limiting case of a single flexible mode, equation (1) becomes equivalent to the normal dynamic response of a single mode. The use of equation (1) insures that a Response Spectra Analysis will always be more conservative than a ZPA static analysis.

The mathematical development of the MRS method is presented in the following sections. The development is presented for systems with no viscous damping in order to simplify the analysis. Since an undamped system is the limiting case of a lightly damped system, the modal response summing rule which applies to the undamped system should also apply to systems with commonly used values of damping, usually less than ten percent of critical.

The first step of the mathematical development is a review of the calculation of response spectra accelerations from single-degree-of-freedom systems. It is shown that the response spectra acceleration can be considered the sum of a "rigid" response equal to the base excitation acceleration and an approximate "dynamic" response.

The next step of the development is a review of a multi-degree-of-freedom system response. The normal solution method is presented and then modified to produce the MRS summation rule. Modal responses are divided into flexible and rigid groupings. Using the results of the single-degree-of-freedom development, the rigid grouping of modal responses are shown to reduce to a simple static analysis. The remaining flexible modal responses are determined using normal formulation with reduced response accelerations.

Results from sample problems are presented following the mathematical development. Both SRSS responses and MRS responses are compared to the time history response.

RESPONSE SPECTRA FORMULATION

The response spectra value for a given frequency is equal to the maximum response to base excitation of a single degree of freedom system with a natural frequency equal to the given frequency. Figure 2 illustrates a single-degree-of-freedom system with a fluctuating base displacement, b(t). The system mass is given by m; the linear spring rate is given by k, and the relative displacement of the mass with respect to the base is given by x(t). For each instant of time, the spring force must equal the inertia force; that is,

$$-kx(t) = m[x''(t) + b''(t)] \tag{2}$$

or, rearranging terms,

$$mx''(t) + kx(t) = -mb''(t) \tag{3}$$

where:

$x''(t)$ = relative acceleration of the mass, and

$b''(t)$ = acceleration of the base.

The base acceleration, b(t), can be represented over a finite interval of time by a Fourier series as

$$b''(t) = a_0 + a_1 \cos(\omega_1 t - \phi_1) + a_2 \cos(\omega_2 t - \phi_2) \ldots + \ a_i \cos(\omega_i t - \phi_i) \tag{4}$$

where:

ω_i = i x (the fundamental frequency of the time interval), and

a_i, ϕ_i = the Fourier coefficients and phase angles of b(t).

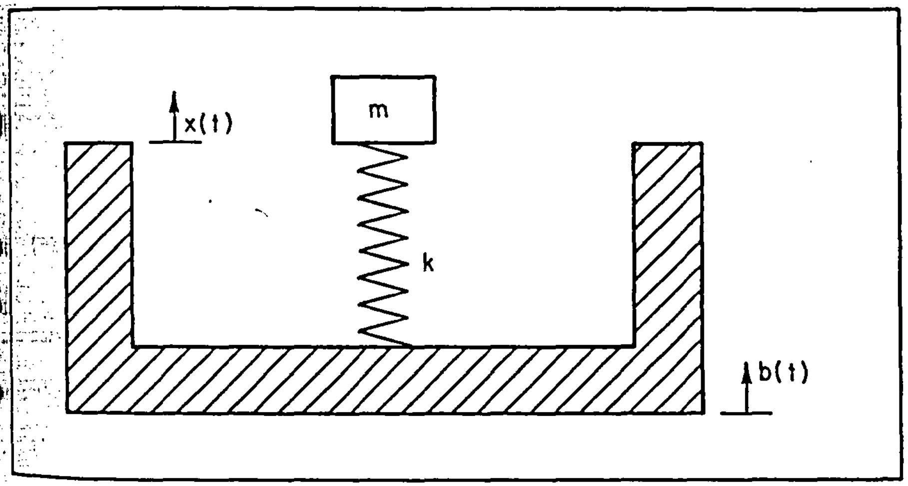

FIG. 2.— Single-Degree-Of-Freedom Oscillator

The steady state solution of equations (3) and (4) for a system initially at rest is given by

$$x(t) = -\frac{a_0}{\omega_n^2} - \frac{a_1\cos(\omega_1 t - \phi_1)}{(\omega_n^2 - \omega_1^2)} - \frac{a_2\cos(\omega_2 t - \phi_2)}{(\omega_n^2 - \omega_2^2)} \cdots - \frac{a_i\cos(\omega_i t - \phi_i)}{(\omega_n^2 - \omega_i^2)} \quad (5)$$

where:

ω_n = the natural frequency of the single-degree-of-freedom system, $(k/m)^{1/2}$.

The "rigid" formulation of equation (3) is given by

$$kx_s(t) = -mb''(t). \quad (6)$$

After substituting equation (4) for $b''(t)$ and ω_n^2 for k/m, equation (6) becomes

$$x_s(t) = -\frac{a_0}{\omega_n^2} - \frac{a_1\cos(\omega_1 t - \phi_1)}{\omega_n^2} \cdots - \frac{a_i\cos(\omega_i t - \phi_i)}{\omega_n^2}. \quad (7)$$

By comparing equations (7) and (5), it can be shown that the relative displacement response can be considered the sum of a "rigid" response, $x_s(t)$, and a "dynamic" response, $x_d(t)$, as

$$x(t) = x_s(t) + x_d(t), \quad (8)$$

where:

$$x_d(t) = -\frac{a_1\cos(\omega_1 t - \phi_1)}{\omega_n^2}\frac{\omega_1^2}{(\omega_n^2 - \omega_1^2)} \cdots - \frac{a_i\cos(\omega_i t - \phi_i)}{\omega_n^2}\frac{\omega_i^2}{(\omega_n^2 - \omega_i^2)}. \quad (9)$$

The acceleration response, A(t), of the single-degree-of-freedom mass is given by the sum of the relative acceleration, $x''(t)$ and the base acceleration, $b''(t)$. Dividing equation (2) by m gives

$$-A(t) = \omega_n^2\, x(t). \quad (10)$$

The undamped acceleration response spectrum value at ω_n is the maximum absolute value of A(t). Utilizing equations (7), (8), and (9), equation (10) can be written as

$$A(t) = A_s(t) + A_d(t) \quad (11)$$

where:

$$A_s(t) = b''(t), \text{ and} \quad (12)$$

$$A_d(t) = a_1\cos(\omega_1 t - \phi_1)\frac{\omega_1^2}{(\omega_n^2 - \omega_1^2)} \cdots + a_i\cos(\omega_i t - \phi_i)\frac{\omega_i^2}{(\omega_n^2 - \omega_i^2)}. \quad (13)$$

Since earthquake excitation is dominated by low frequency components, $A_d(t)$ becomes small as ω_n becomes large. An acceleration response spectra becomes constant above the cut-off frequency because, for rigid frequencies, the peak value of $A_d(t)$ is negligible compared to the peak value of $A_s(t)$.

Since $A_s(t)$ and $A_d(t)$ each vary randomly in time, their respective maximum values are not likely to occur simultaneously. The maximum magnitude of $A_s(t)$ is given by the ZPA of an acceleration response spectrum. The value of the acceleration response spectrum represents the maximum magnitude of A(t), (A(t), maximum). Assuming that A(t), maximum is approximated by an SRSS combination of $A_s(t)$, maximum and $A_d(t)$, maximum, then an expression for the maximum magnitude of $A_d(t)$ is given by

$$A_d(t),\ \text{maximum} = [(A(t),\ \text{maximum})^2 - (ZPA)^2]^{1/2}. \tag{14}$$

For rigid modes, A(t), maximum, is equal to the ZPA, and $A_d(t)$, maximum is equal to zero. Equations (11), (12), and (14) will be used for development of multi-degree-of-freedom systems in the following sections.

MULTI-DEGREE OF FREEDOM RESPONSE

Equation (3), written in matrix form for an n degree-of-freedom-system, is

$$\underset{(n\times n)}{\lceil m \rfloor} \underset{(n\times 1)}{\{x''(t)\}} + \underset{(n\times n)}{[\,k\,]} \underset{(n\times 1)}{\{x(t)\}} = -b''(t) \underset{(n\times n)}{\lceil m \rfloor} \underset{(n\times 1)}{\{1\}} \tag{15}$$

where:

$\lceil m \rfloor$ is the diagonal mass matrix,

$[\,k\,]$ is the symmetric stiffness matrix,

$\{x(t)\}$ is the relative displacement vector,

$b''(t)$ is the base acceleration, and

$\{1\}$ is a column vector of 1's.

The equations represented by expression (15) can be de-coupled by premultiplying by a transposed matrix of n orthogonal eigenvectors and making the following substitutions:

$$\underset{(n\times n)}{\lceil M \rfloor} = \underset{(n\times n)}{[\,\Phi\,]}^T \underset{(n\times n)}{\lceil m \rfloor} \underset{(n\times n)}{[\,\Phi\,]} \tag{16}$$

$$\underset{(n\times n)}{\lceil K \rfloor} = \underset{(n\times n)}{[\,\Phi\,]}^T \underset{(n\times n)}{[\,k\,]} \underset{(n\times n)}{[\,\Phi\,]} \tag{17}$$

$$\underset{(n\times 1)}{\{x(t)\}} = \underset{(n\times n)}{[\,\Phi\,]} \underset{(n\times 1)}{\{q(t)\}} \tag{18}$$

where

$[\,\Phi\,]$ = a matrix composed of n orthogonal eigenvectors of expression (15),

$[\,\Phi\,]^T = [\,\Phi\,]$, transpose, and

$\{q(t)\}$ = generalized relative deflections.

Substituting expressions (16), (17) and (18) into expression (15) yields

$$\underset{(n\times n)}{\lceil M \rfloor} \underset{(n\times 1)}{\{q''(t)\}} + \underset{(n\times n)}{\lceil K \rfloor} \underset{(n\times 1)}{\{q(t)\}} = -b''(t) \underset{(n\times n)}{[\,\Phi\,]}^T \underset{(n\times n)}{\lceil m \rfloor} \underset{(n\times 1)}{\{1\}}\ . \tag{19}$$

Expression (19) represents a set of n independent, single-degree-of-freedom equations. Each equation in expression (19) is equivalent to a single-degree-of-freedom system subjected to a base acceleration equal to $b''(t)$ multiplied by a factor, Γ_i. The solution to each equation for a system initially at rest is

$$q_i(t) = \frac{\Gamma_i\, A_i(t)}{\omega_i^2}. \tag{20}$$

where

$$\Gamma_i = \frac{\underset{(1\times n)}{[\,\Phi_i\,]}^T \underset{(n\times n)}{\lceil m \rfloor} \underset{(n\times 1)}{\{1\}}}{M_i} \tag{21}$$

$$\omega_i^2 = K_i/M_i,\ \text{and} \tag{22}$$

$A_i(t)$ = the absolute acceleration response at ω_i.

Insertion of equation (20) into expression (18) yields that each

$$x_i(t) = \phi_{i1}\frac{\Gamma_1 A_1(t)}{\omega_1^2} + \phi_{i2}\frac{\Gamma_2 A_2(t)}{\omega_2^2} \ldots + \phi_{in}\frac{\Gamma_n A_n(t)}{\omega_n^2} \; . \tag{23}$$

In a Response Spectra Modal Analysis only the peak value of each $A_i(t)$ in equation (23) is known. Since all $A_i(t)$ may not peak at the same time, the peak value of $x_i(t)$ must be approximated by some combination of the terms in equation (23) when each term is evaluated at its maximum value.

The most common method used to combine the responses in equation (23) is by SRSS. An inherent assumption of the SRSS method is that each term in equation (23) will peak randomly in time. A more refined estimate of $x_i(t)$ can be obtained by grouping into a single term the parts of equation (23) which peak as the base excitation peaks. The new grouping can be achieved by the steps that follow.

Expression (18) is separated into flexible and rigid portions as

$$\underset{(n\times 1)}{\{x(t)\}} = \underset{(n\times \ell)}{\lfloor \phi_f \rfloor} \underset{(\ell\times 1)}{\{q_f(t)\}} + \underset{(n\times j)}{\lfloor \phi_r \rfloor} \underset{(j\times 1)}{\{q_r(t)\}} \tag{24}$$

where:

ℓ = number of flexible modes,

j = number of rigid modes, ($j+\ell = n$),

$\lfloor \phi_f \rfloor$ = a matrix composed of the ℓ flexible, orthogonal eigenvectors,

$\lfloor \phi_r \rfloor$ = a matrix composed of the j, rigid orthogonal eigenvectors,

$\{q_f(t)\}$ = a vector of the ℓ flexible, generalized deflections, and

$\{q_r(t)\}$ = a vector of the j rigid, generalized deflections.

Substitution of equation (20) into expression (24) yields

$$\underset{(n\times 1)}{\{x(t)\}} = \underset{(n\times \ell)}{\lfloor \phi_f \rfloor} \underset{(\ell\times 1)}{\left\{\frac{\Gamma_f A_f(t)}{\omega_f^2}\right\}} + \underset{(n\times j)}{\lfloor \phi_r \rfloor} \underset{(j\times 1)}{\left\{\frac{\Gamma_r A_r(t)}{\omega_r^2}\right\}} \tag{25}$$

where:

$\left\{\frac{\Gamma_f A_f(t)}{\omega_f^2}\right\}$ = a column vector composed of the ℓ values of $\Gamma_i A_i(t)/\omega_i^2$ for the flexible modes, and

$\left\{\frac{\Gamma_r A_r(t)}{\omega_r^2}\right\}$ = a column vector composed of the j values of $\Gamma_i A_i(t)/\omega_i^2$ for the rigid modes.

After substituting equation (11) and noting that $A_d(t)$ is zero for rigid modes, expression (25) becomes

$$\underset{(n\times 1)}{\{x(t)\}} = \underset{(n\times \ell)}{\lfloor \phi_f \rfloor} \underset{(\ell\times 1)}{\left\{\frac{\Gamma_f A_s(t) + \Gamma_f A_{f,d}(t)}{\omega_f^2}\right\}} + \underset{(n\times j)}{\lfloor \phi_r \rfloor} \underset{(j\times 1)}{\left\{\frac{\Gamma_r A_s(t)}{\omega_r^2}\right\}} \tag{26}$$

where:

$A_s(t)$ = the "rigid" value of the absolute acceleration response (independent of modal frequency), and

$A_{f,d}(t)$ = the "dynamic" value of the absolute acceleration response for each flexible mode, (dependent on the frequency of the mode).

Since, $\ell+j = n$, expression (26) can be expressed as

$$\underset{(n\times 1)}{\{x(t)\}} = \underset{(n\times n)}{[\Phi]} \underset{(n\times 1)}{\left\{\frac{\Gamma_i}{\omega_i^2}\right\}} A_s(t) + \underset{(n\times \ell)}{[\Phi_f]} \underset{(\ell\times 1)}{\left\{\frac{\Gamma_f A_{f,d}(t)}{\omega_f^2}\right\}} . \qquad (27)$$

The coefficient of $A_s(t)$ in equation (27) can be evaluated by the steps that follow.

Using equation (20) and expression (18), the solution for a multi-degree-of-freedom system with an absolute acceleration response $A_i(t)$ at frequency ω_i can be expressed as

$$\{x(t)\} = \underset{(n\times n)}{[\Phi]} \underset{(n\times 1)}{\left\{\frac{\Gamma_i A_i(t)}{\omega_i^2}\right\}} . \qquad (28)$$

If a constant acceleration, G, is applied to the structure, then the absolute acceleration response at each frequency is equal to G, and equation (28) becomes

$$\{x\} = \underset{(n\times n)}{[\Phi]} \underset{(n\times 1)}{\left\{\frac{\Gamma_i}{\omega_i^2}\right\}} G. \qquad (29)$$

For a constant acceleration, G, the value of $\{x(t)\}$ can also be determined from a static analysis where

$$\{x\} = \underset{(n\times n)}{[k]}^{-1} \underset{(n\times n)}{\lceil m \rfloor} \underset{(n\times 1)}{\{1\}} G . \qquad (30)$$

Equating expressions (29) and (30) shows that

$$\underset{(n\times n)}{[\Phi]} \underset{(n\times 1)}{\left\{\frac{\Gamma_i}{\omega_i^2}\right\}} = \underset{(n\times n)}{[k]}^{-1} \underset{(n\times n)}{\lceil m \rfloor} \underset{(n\times 1)}{\{1\}} . \qquad (31)$$

Substitution of expression (31) into expression (27) produces the final result, that is

$$\underset{(n\times 1)}{\{x(t)\}} = \underset{(n\times n)}{[k]}^{-1} \underset{(n\times n)}{\lceil m \rfloor} \underset{(n\times 1)}{\{1\}} A_s(t) + \underset{(n\times \ell)}{[\Phi_f]} \underset{(\ell\times 1)}{\left\{\frac{\Gamma_f A_{f,d}(t)}{\omega_f^2}\right\}}. \qquad (32)$$

Equation (32) represents a re-grouping of equation (23). The response given by the first term of equation (32) will peak as the base excitation peaks. It can be considered the rigid mode of the system which vibrates in phase with the base.

The second term of equation (32) represents the dynamic contributions of the flexible modes. It has the same form as the normal solution for multi-degree-of-freedom systems given by equations (20) and (23). The only differences are that only flexible modes are included in the summation, and the response accelerations used are approximated by equation (14). Each of the dynamic contributions can be considered to vibrate randomly in time.

Equation (32) has been regrouped into a form where each term vibrates independently. An SRSS combination of the terms can be applied to approximate the maximum system response. The resulting MRS summation method is given by equation (1).

Examples of the MRS method resulting from a square root sum of the squares combination of the new grouping are presented in the following section.

EXAMPLE, HORIZONTAL PIPE RESTRAINT

The horizontal pipe restraint shown in Figure 3 is used to demonstrate the modified response summation method. Eight parallel pipes, each weighing 386.4 lbf. (1719 N), are restrained by a single tapered member. The analytical model is presented in Figure 4. Three different models are created by varying the stiffness, K, of the restraint as 5000 lbf/in (876 N/mm), 10 000 lbf/in (1752 N/mm), and 20 000 lbf/in (3504 N/mm). The models are referred to as 1, 2, and 3, respectively.

The stiffness of model 2 is selected to produce an equal number of flexible and rigid modes of vibration. Model 1 is more flexible than model 2 while model 3 is more rigid. The natural frequencies of each model are given in Table 1.

Each analytical model has been subjected to the north/south component of the 1940 El Centro earthquake. A time history analysis as well as MRS and SRSS Response Spectra Modal Analyses were performed for each model. The SRSS summation included all modes. The El Centro response spectra for 2 percent critical damping was used in each response spectra analysis. Each mode in the time history analysis was given 2 percent critical damping by use of a damping

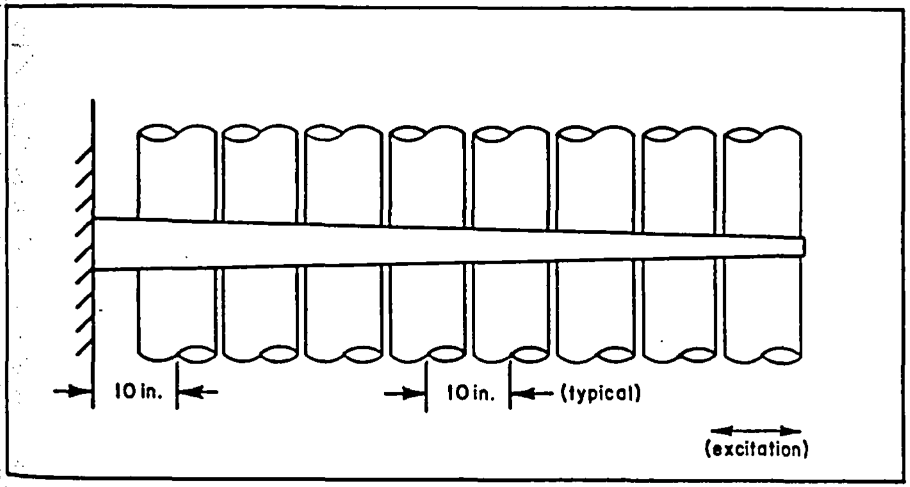

FIG. 3.— Horizontal Pipe Restraint System Example (lin. = 25.4 mm)

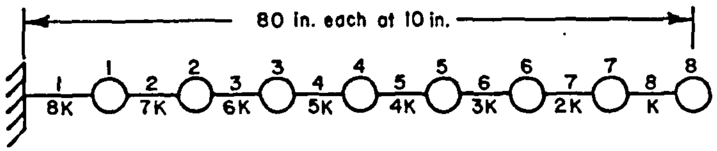

FIG. 4.— Analytical Model of the Pipe Restraint System (1in. = 25.4 mm)

matrix composed of terms generated from a Caughey series (1). The acceleration response spectrum value for each frequency is included in Table 1. ZPA acceleration for the response spectra was taken as .33 G starting at 34 Hz.

Results from the analyses of the three models are tabulated in Tables 2 and 3. Acceleration responses of the pipes are given in Table 2. Axial loads in the pipe restraint are given in Table 3. The results of model 2 are typical and are plotted as fraction of time history response in Figures 5 and 6. It should be noted that the fraction of time history acceleration response at fixed points is identically 1 for an MRS summation and identically 0 for an SRSS summation.

DISCUSSION OF THE RESULTS

An examination of the results shows that the MRS predictions of response are, in general, comparable or better than the SRSS predictions when compared to time history response. Both methods predict restraint load responses more accurately than they predict pipe acceleration responses. The acceleration responses of the SRSS method prove to be more biased by fixed points than those generated by the MRS method. The SRSS predictions for the most flexible model show slightly better results than the MRS predictions at points far away from the fixed end of the restraint. As the system becomes more rigid, the predictions of the SRSS method become less accurate. The predictions of the MRS method show greater stability over the entire frequency range.

PRACTICAL APPLICATIONS

The intent of the MRS method is to reduce the required computation while increasing the accuracy of modal summation predictions. The MRS method

TABLE 1.--Natural Frequencies of Models 1, 2, and 3, and the Corresponding Response Spectra Accelerations From the North/South Component of the 1940 El Centro Earthquake

Mode Number	Model 1 Frequency (Hertz)	Model 1 Acceleration (g)	Model 2 Frequency (Hertz)	Model 2 Acceleration (g)	Model 3 Frequency (Hertz)	Model 3 Acceleration (g)
1	4.6	1.13	6.6	1.41	9.3	.61
2	10.7	.58	15.1	.46	21.4	.38
3	16.9	.42	23.9	.37	33.8	.33
4	23.2	.37	32.9	.39	46.5	.33
5	29.9	.38	42.2	.33	59.7	.33
6	36.9	.33	52.2	.33	73.8	.33
7	44.7	.33	63.1	.33	89.3	.33
8	53.8	.33	76.1	.33	107.6	.33

TABLE 2.--Peak Absolute Acceleration Response in g's at Each Node of the Pipe Restraint Models

Response Method	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8
(a) Model 1								
Time history	.398	.490	.636	.777	.942	1.143	1.397	1.852
SRSS	.193	.361	.536	.723	.923	1.142	1.392	1.712
% Time history	49	74	84	93	98	100	100	92
MRS	.364	.458	.591	.750	.930	1.132	1.364	1.657
% Time history	92	94	93	97	99	99	98	90
(b) Model 2								
Time history	.354	.521	.717	.930	1.173	1.423	1.685	1.986
SRSS	.214	.413	.629	.867	1.128	1.414	1.734	2.105
% Time history	61	79	88	93	96	99	103	106
MRS	.376	.500	.677	.889	1.133	1.405	1.712	2.060
% Time history	106	96	94	96	97	99	102	104
(c) Model 3								
Time history	.308	.327	.406	.477	.541	.625	.713	.840
SRSS	.135	.222	.311	.407	.509	.623	.756	.932
% Time history	44	68	77	85	94	100	106	111
MRS	.337	.359	.398	.453	.522	.604	.702	.825
% Time history	109	110	98	95	96	97	98	98

TABLE 3.--Peak Axial Load in Pounds at Each Spring of the Pipe Restraint Models

Response Method	Spring 1	Spring 2	Spring 3	Spring 4	Spring 5	Spring 6	Spring 7	Spring 8
				(a) Model 1				
Time history	2608	2541	2403	2189	1953	1656	1254	715
SRSS	2575	2518	2398	2211	1951	1613	1188	662
% Time history	99	99	100	101	100	97	95	93
MRS	2663	2565	2416	2208	1934	1588	1161	640
% Time history	102	101	101	101	99	96	93	90
				(b) Model 2				
Time history	3373	3240	3039	2762	2403	1955	1411	765
SRSS	3211	3141	2994	2763	2439	2014	1476	813
% Time history	95	97	99	100	102	103	105	106
MRS	3282	3180	3009	2761	2425	1994	1454	796
% Time history	97	98	99	101	101	102	103	104
				(c) Model 3				
Time history	1551	1456	1333	1181	1005	804	586	325
SRSS	1384	1352	1286	1185	1046	866	640	360
% Time history	89	93	97	100	104	108	109	111
MRS	1539	1439	1319	1179	1013	818	589	319
% Time history	99	99	99	100	101	102	101	98

Note: 1 lbf = 4.45N

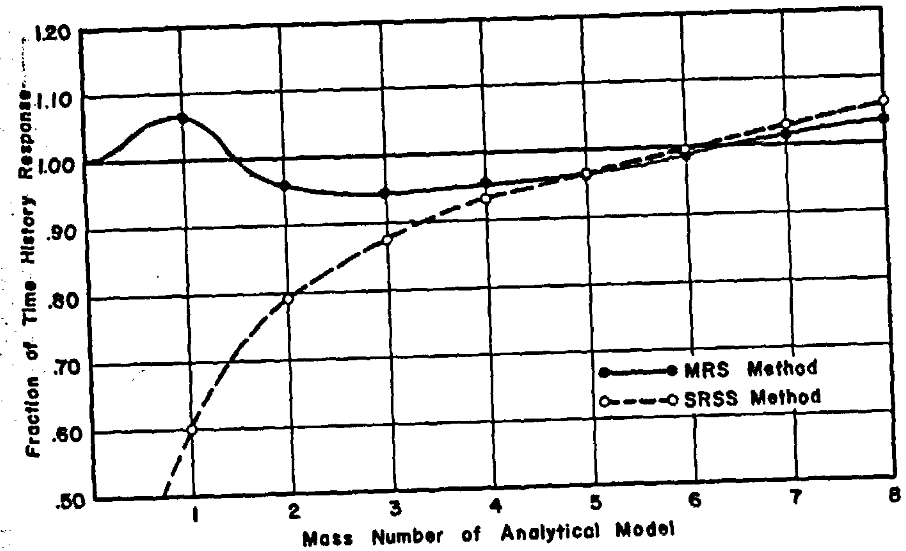

FIG. 5.— Absolute Acceleration Response For Pipes of Model 2.

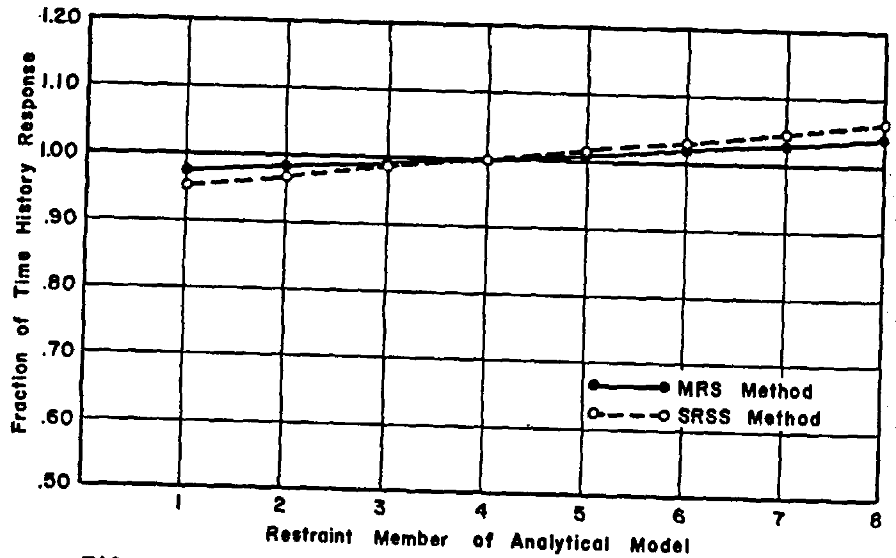

FIG. 6.— Axial Load Response For Restraint Members of Model 2.

presented above can be implemented without modification of most existing, general purpose, finite element computer codes. The need to determine the significance of high frequency modes is eliminated. However, if a system contains many flexible modes, the determination of modal significance is still required. Suggested additional work would include developing a quantitative definition of modal significance.

CONCLUSIONS

A modified response summation rule, MRS, has been proposed as an alternative to the SRSS rule for combining modal responses from a Response Spectra Modal Analysis. The MRS method reduces the cost of the analysis because only flexible modes are required. It increases the certainty of results by eliminating the need for selection of significant high frequency modes and allowing for simultaneous rigid response. Comparisons of the MRS method and the SRSS method with time history response show that the MRS method is, in general, more reliable than the SRSS method.

APPENDIX I.-REFERENCES

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APPENDIX II.- NOTATION

The following symbols are used in this paper:

a = coefficient of a Fourier series;
A = absolute acceleration response of a mode;
$b(t)$ = time dependent base displacement;
G = arbitrary constant acceleration;
k = spring stiffness;
K = generalized spring stiffness;
m = mass;
M = generalized mass;
MRS = modified response summation;
$q(t)$ = time dependent generalized relative displacement;
R = response quantity;
SRSS = square root sum of squares response summation;
t = time;
$x(t)$ = time dependent relative displacement;
ZPA = zero period acceleration of response spectra;
Γ = modal participation factor;
ϕ = phase angle of a Fourier series;
ϕ = mode shape eigenvector;
ω = radian frequency;
" = twice differentiated with respect to time;
$\{\}$ = column vector;
$[\,]$ = matrix;
$\lceil\ \rfloor$ = diagonal matrix;
(x) = matrix dimensions.

Subscripts

d = dynamic portion of a response;

f = flexible quantity (natural frequency below the cut-off frequency);

i = counting index;

j = number of rigid modes;

n = total number of degrees of freedom;

r = rigid quantity (natural frequency above the cut-off frequency);

s = quantity based on "rigid" formulation;

n = natural frequency;

$1,2,3\ldots$ = i^{th} term of Fourier series.

Superscripts

T = matrix transpose.

Uncertainties In Seismic Design Of Equipment Mounted Subsystems[a]

By L. D. Gerdes[1], and R. P. Kassawara[2]

INTRODUCTION

Purpose

This paper investigates the conservatism associated with the accounting for uncertainties in system properties by artificial seismic response spectrum peak broadening at intermediate steps of a properly executed series of substructure analyses. The substructures involved are the containment building and the large, massive Reactor Coolant System (RCS). The uncertainties of concern are primarily those associated with the knowledge of the frequencies of the reinforced concrete containment internal structure. The focus of this paper is on the effects that this process of peak broadening has on light, uncoupled equipment attached to the RCS. The effects of intermediate peak broadening are compared to those of performing a single coupled model analysis of the building - RCS and then broadening peaks at the conclusion of the analysis. Comparisons are also made to the procedure of actually varying the parameters of the building model to account for uncertainties without broadening peaks at all. It is shown that the latter two procedures are reasonable methods for accounting for uncertainties in building response and that they yield similar results while the procedure of intermediate peak broadening can be unreasonably conservative and, in some instances, unconservative.

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